EC5104/5104R: Mathematics for Economists

Semester 1, AY 2022-23

Instructor

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Time and Location

Tuesday, 9am-1pm (AS4-06-04) and Friday, 9am-11am (room to be announced). Please note that although the time slot for Tuesday is for 4 hours, it's pointless to talk about/listen to maths for 4 hours. I'll spend at most 2 and ½ hours in lecture on Tuesdays starting at 9am, and the remaining time we'll make up on Fridays in tutorial. *You should be available to attend both sessions in order to enrol in the module.* The online registration system won't recognize Friday tutorials, but you need to take the arrangement into account.

Office Hours

Dept lounge/SJR as and when it is mutually convenient.

Textbooks

No specific textbook is assigned. Some of the materials can be found with varied emphasis in the following textbooks:

- Simon, C. and L. Blume, *Mathematics for Economists*, 1994
- Sundaram. R.K., *A First Course in Optimization Theory*, 1996
- Ok, E., *Real Analysis with Economic Applications*, 2007

I'll draw upon ***Lecture Notes*** (*Lectures on Mathematical Analysis for Economists*, 2011) written by Prof. Tapan Mitra (Cornell University).

Objective

Introduce students to basic maths needed for economics PhD training.

Grading

Mid-term exam (30%), CA (30%, based on tutorials), and final exam (40%).

Mid-term exam will be held during regular lecture hours, in week 7. The final exam is comprehensive. Both exams are closed-book.

Tutorial assignments will not be graded until after the Final exam has been graded. And only random selections will be looked at. The grades are usually determined based on how seriously students are attempting the problem sets rather than just the accuracy. You are free to consult each other, but please write your own solutions.

Tutorials

Students assigned for tutorials will solve the exercises in class. The class/tutorial will be on a different day from the day of the lecture. Please mark your calendar as tutorial attendance is compulsory. Maths (for Economics) can only be learnt by a sink-and-swim approach, by solving the problem sets. I advise strongly that you take the tutorial exercises seriously.

Lectures

A subset of the following topics will be covered. We will not necessarily cover the topics in the stated order.

Topic I. Linear Algebra

VECTORS:
Vector Spaces, Linear Dependence of Vectors, Rank and Basis, Inner Product and Norm

MATRICES:
Matrix Operations, Transpose of a Matrix, Rank of a Matrix, Inverse of a Matrix, Relationship Between Invertible and Non-Singular Matrices

SIMULTANEOUS LINEAR EQUATIONS:
System of Linear Equations, Existence of Solutions, Uniqueness of Solutions, Calculation of Solutions, Determinants, Matrix Inversion, Cramer's Rule

CHARACTERISTIC VALUES and VECTORS:
The Characteristic Value Problem; Characteristic Values, Trace and Determinant of a Matrix; Characteristic Values and Vectors of Symmetric Matrices; Spectral Decomposition of Symmetric Matrices; Quadratic Forms; Characterization of Quadratic Forms; Alternative Characterization of Quadratic Forms

Topic II. Real Analysis

BASIC CONCEPTS:
Norm and Distance, Open and Closed Sets, Convergent Sequences, Compact Sets, Continuous Functions, Existence of Solutions to Constrained Optimization Problems

DIFFERENTIAL CALCULUS (to be self-taught):
Partial Derivatives, Composite Functions and the Chain Rule, Homogeneous Functions and Euler's Theorem, The Inverse and Implicit Function Theorems

CONVEX ANALYSIS:
Convex Sets, Separating Hyperplane Theorem for Convex Sets, Continuous and Differentiable Functions on Convex Sets, Concave Functions, Quasi-Concave Functions

Topic III. Classical Optimization Theory

UNCONSTRAINED OPTIMIZATION:
Necessary Conditions for a Local Maximum, Sufficient Conditions for a Local Maximum, Sufficient Conditions for a Global Maximum, The Method of Least Squares, The Envelope Theorem

CONSTRAINED OPTIMIZATION:
Necessary Conditions for a Constrained Local Maximum, The Arithmetic Mean-Geometric Mean Inequality, Sufficient Conditions for a Constrained Local Maximum, Sufficient Conditions for a Global Maximum

Topic IV. Modern Optimization Theory

CONCAVE PROGRAMMING:
Constrained Global Maxima and Saddle Points, The Kuhn-Tucker Conditions and Saddle Points, The Kuhn-Tucker Conditions and Constrained Local Maxima, Constrained Local and Global Maxima,

QUASI-CONCAVE PROGRAMMING:
Concave Functions, Quasi-concave functions, The Sufficiency Theorem of Arrow-Enthoven, The Necessity Theorem of Arrow-Enthoven

Topic V. Linear Programming

The Primal and Dual Problems, Optimality Criterion, The Basic Duality Theorems, Complementary Slackness